

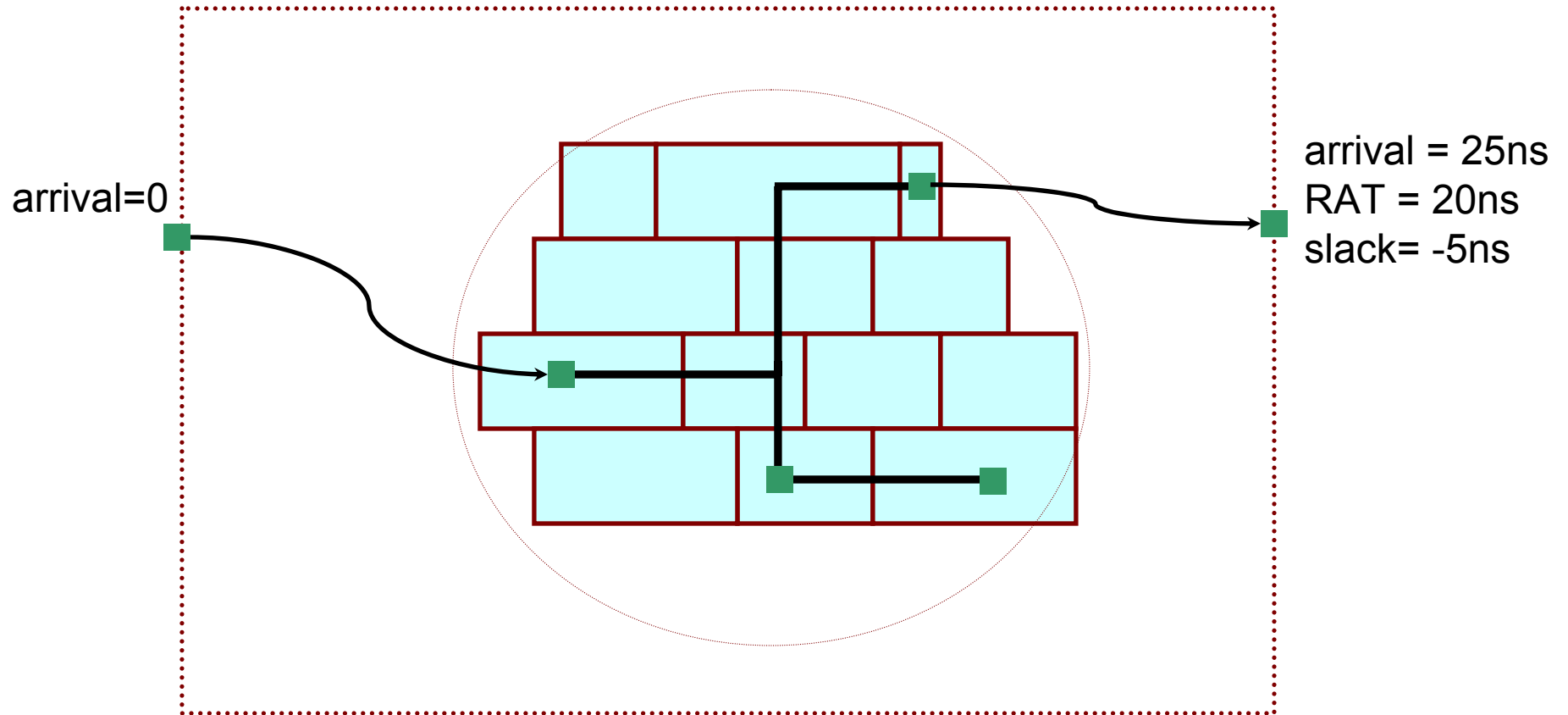
# Physical Design of Digital Integrated Circuits (EN0291 S40)

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# Lecture 10: Repeater (Buffer) Insertion

- Introduction to Buffering
- Buffer Insertion
  - Two-pin nets
  - Multi-pin nets

# Big picture: when is buffering needed?

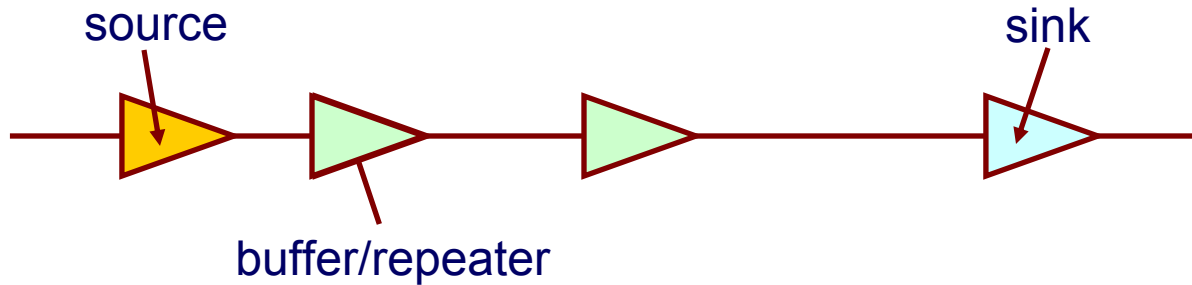


Placement → Steiner tree construction → STA  
Timing is not met! What are you going to do?



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# Buffer insertion in the simple case of two-pin wires

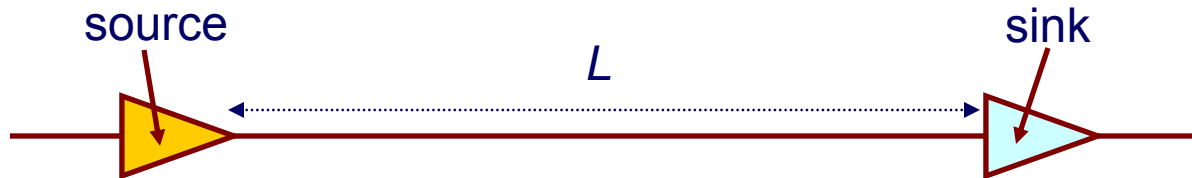


Q: How to improve the delay from source to sink? → Insert repeaters

## Two questions:

- What is the position that minimizes the delay?
- How many repeaters to insert to minimize the delay?

# What is the delay from source to sink without any repeaters?



Delay from source to sink:

$$D_0 = D_{src} + R_{src}(cL + C_{snk}) + rL\left(\frac{cL}{2} + C_{snk}\right)$$

$D_{src}$ : intrinsic delay of source

$R_{src}$ : effective output resistance

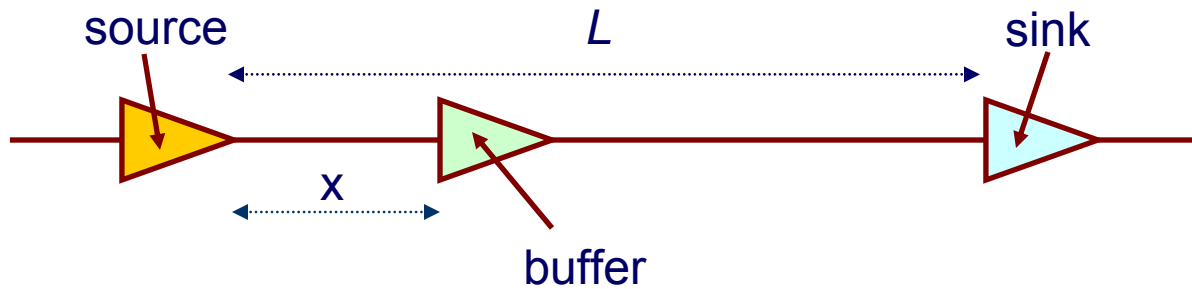
$r/c$ : resistance/capacitance per unit length

$L$ : wire length

$C_{snk}$ : input capacitance of sink



If you have one repeater, where is the optimal position to insert it?



Delay from source to sink:

$$D_1(x) = D_{src} + R_{src}(cx + C_{buf}) + rx\left(\frac{cx}{2} + C_{bf}\right) + D_{bf} + R_{bf}(c(L - x) + C_{snk}) + R(L - x)\left(\frac{c(L - x)}{2} + C_{snk}\right)$$

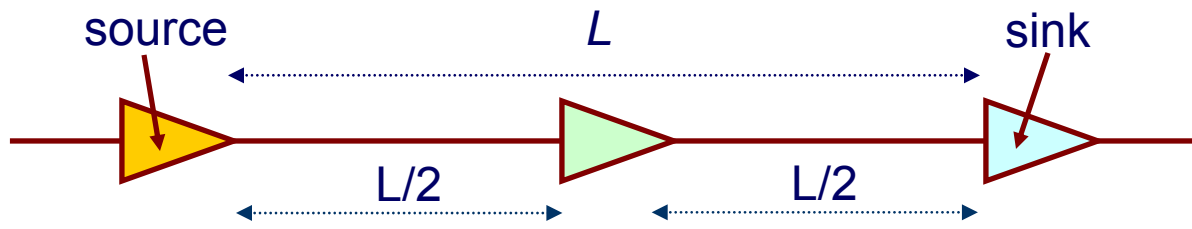
Minimum delay is attained when

$$\frac{dD_1(x)}{dx} = 0 \text{ at } x = \frac{L}{2} + \frac{R_{buf} - R_{snk}}{2r} + \frac{C_{snk} - C_{buf}}{2c}$$

Makes sense to add a buffer only if  $D_0 - D_1 > 0$



If there are multiple buffers, where are the optimal locations to insert them?

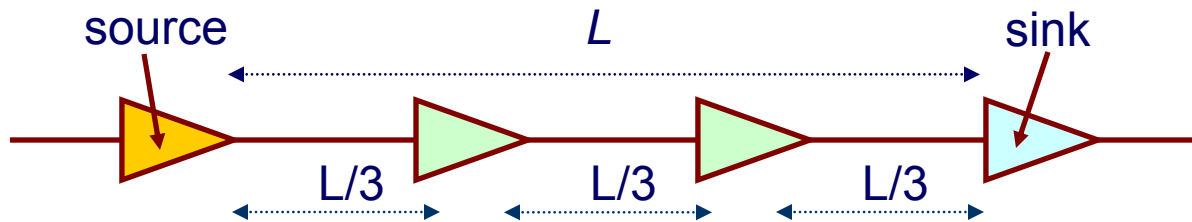


$$\frac{dD_1(x)}{dx} = 0 \text{ at } x = \frac{L}{2} + \frac{R_{buf} - R_{snk}}{2r} + \frac{C_{snk} - C_{buf}}{2c}$$

If  $R_{buf} = R_{snk}$  and  $C_{snk} = C_{buf}$   
then the optimal location for the buffer is at distance  $L/2$

If there are  $k$  buffers then minimum delay will occur when they are equally spaced, i.e., separation distance is  $L/(k+1)$

# How many repeaters can we keep adding to minimize the delay?



Same analysis as before!

Delay from source to sink:

$$\begin{aligned} D_2(x) = & D_{src} + R_{src} \left( \frac{cL}{3} + C_{buf} \right) + \frac{rL}{3} \left( \frac{cL}{6} + C_{bf} \right) \\ & + D_{bf} + R_{bf} \left( \left( \frac{cL}{3} + C_{bf} \right) + \frac{RL}{3} \left( \frac{cL}{6} + C_{bf} \right) \right) \\ & + D_{bf} + R_{bf} \left( \frac{cL}{3} + C_{snk} \right) + \frac{RL}{3} \left( \frac{cL}{6} + C_{snk} \right) \end{aligned}$$

If  $D_2 < D_1$  then use two buffers

Keep on adding buffers until  $D_{k+1} \geq D_k$  then the optimal number of buffers is  $k$  (see lecture readings for a closed form of  $k$ )



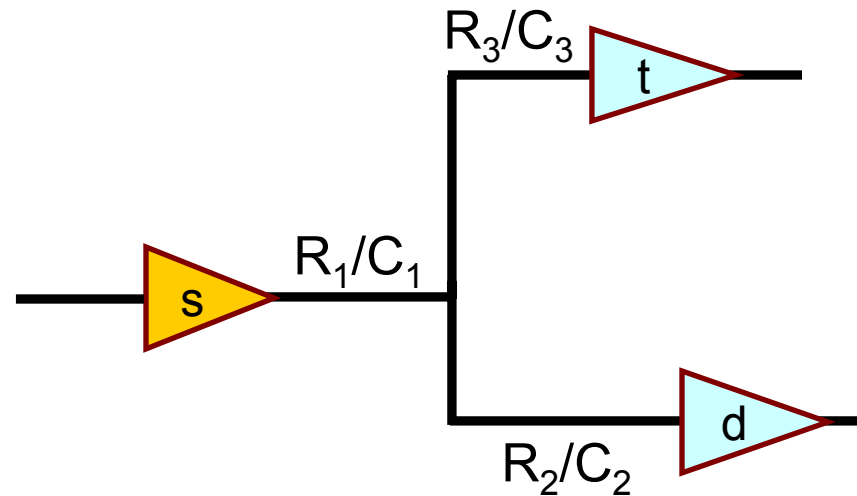


# Lecture 10: Repeater (Buffer) Insertion

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# Recap: How to calculate pin-to-pin delay in multi-pin trees?



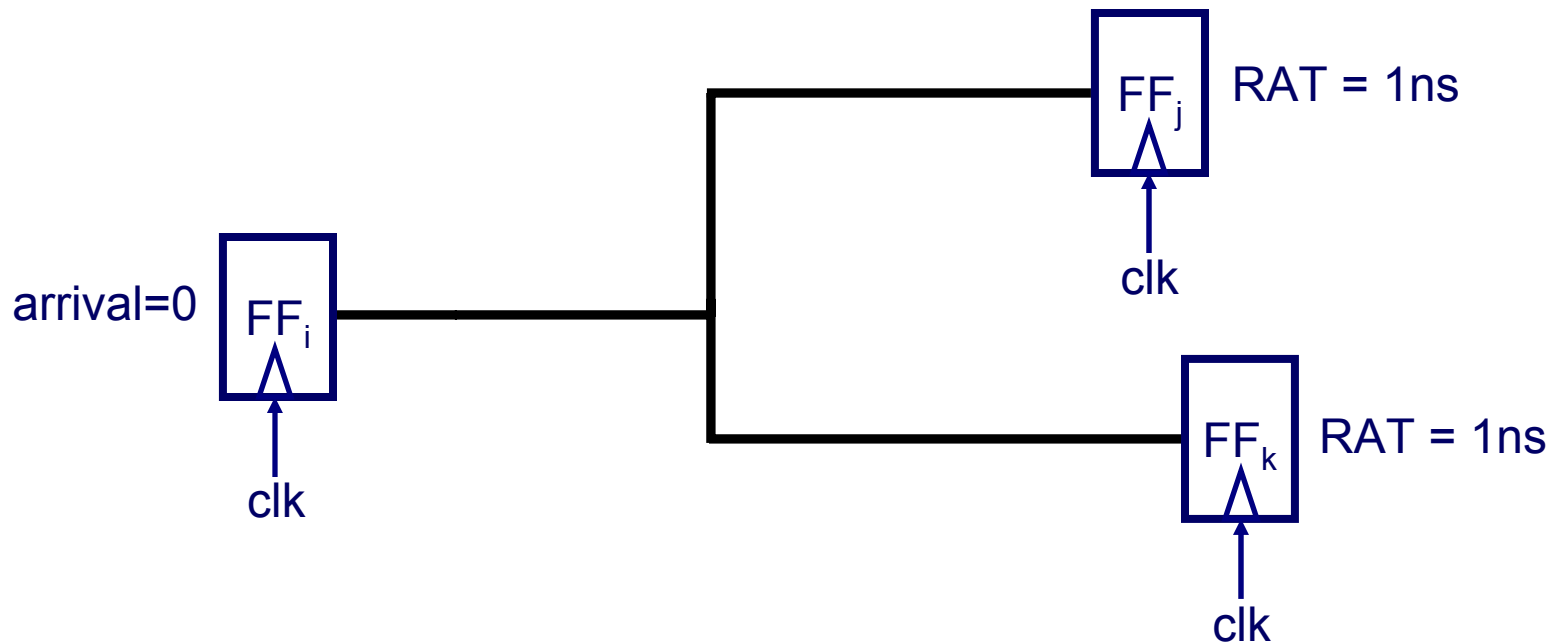
Delay from  $s \rightarrow t$ :

$$D_s + R_s(C_1 + C_3 + C_2 + C_t + C_d) \\ + R_1(C_1/2 + C_2 + C_3 + C_t + C_d) + R_3(C_3/2 + C_t)$$

Delay from  $s \rightarrow d$ :

$$D_s + R_s(C_1 + C_3 + C_2 + C_t + C_d) \\ + R_1(C_1/2 + C_2 + C_3 + C_t + C_d) + R_2(C_2/2 + C_d)$$

# Repeater insertion in trees



Suppose this is a simple circuit that is required to operate at 1GHz

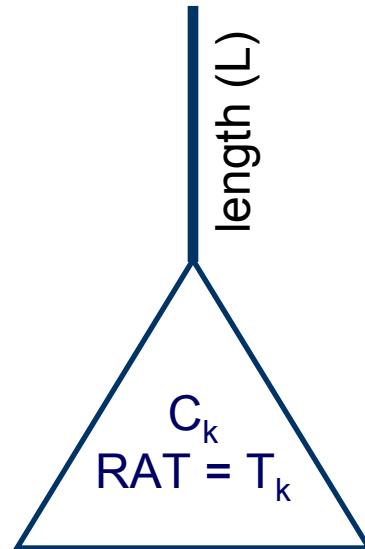
Assume the **delay from i to j** is 1.1ns and the **delay from i to k** is 1.2ns

Therefore the slack at the source is  $\min(1-1.2, 1-1.1) = -0.2$

Our objective is to **maximize** the slack at the source (hint: minimize the delay from source to sinks via repeater insertion)

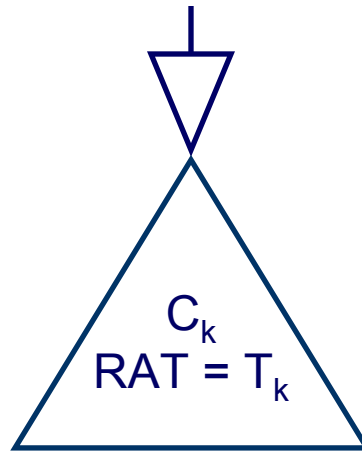
# Basic operations: adding a wire

$$\text{load} = C_k + cL$$
$$\text{slack} = T_k - rLC_k - \frac{rcL^2}{2}$$



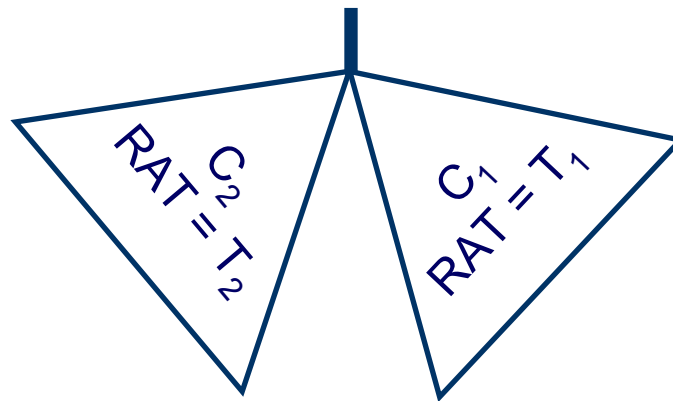
# Basic operations: adding a buffer

$$\text{load} = C_{buf}$$
$$\text{slack} = T_k - RC_k - D_{buf}$$



# Basic operations: merging two subtrees

$$\text{load} = C_1 + C_2$$
$$\text{slack} = \min(T_1, T_2)$$



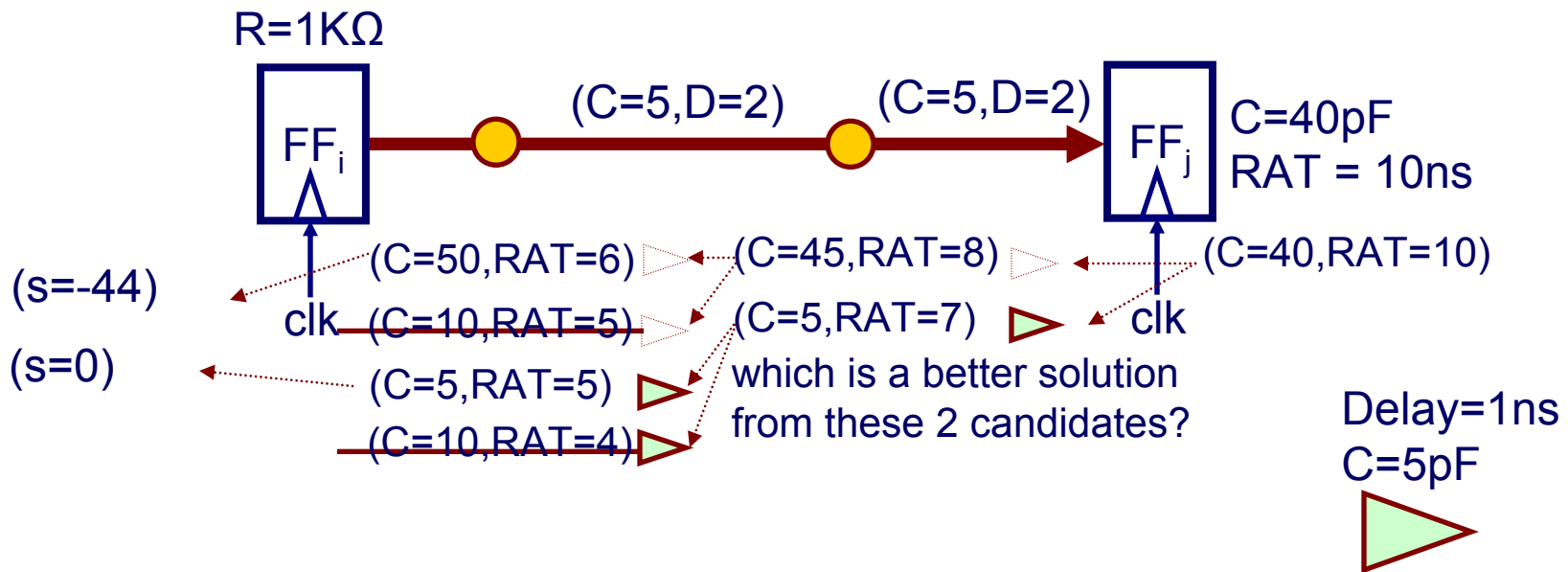
# Generating candidates: the buffer case

R: Effective output resistance,

C: capacitance (input pin/wire)

RAT: Required Arrival Time for the circuit to work according specifications

D: delay



Rule  $(C_2, R_2)$  is considered redundant candidate if

There exists some other candidate  $(C_1, R_1)$  such  $R_1 \geq R_2$  and  $C_1 \leq C_2$

→ Adding a buffer can only increase the candidate list size by 1



# Generating candidates: the merging case

## Candidates of subtree 1

$(C_{11}, RAT_{11})$

$(C_{21}, RAT_{21})$

$(C_{..}, RAT_{..})$

$(C_{n1}, RAT_{n1})$

$RAT_{11} > RAT_{21} > .. > RAT_{n1}$

$C_{11} < C_{21} < ... < C_{2n}$

## Candidates of subtree 2

$(C_{12}, RAT_{12})$

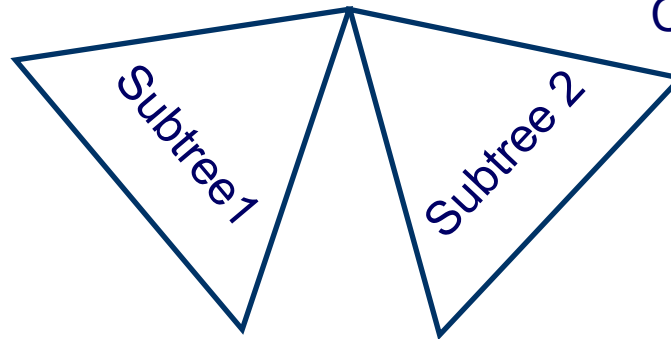
$(C_{22}, RAT_{22})$

$(C_{..}, RAT_{..})$

$(C_{m2}, RAT_{m2})$

$RAT_{12} > RAT_{22} > .. > RAT_{m2}$

$C_{12} < C_{22} < ... < C_{m2}$



What is the maximum number of candidates in the newly merged subtree?

At most  $n+m$ . For every RAT value, we only need to keep the candidate with the least capacitance!

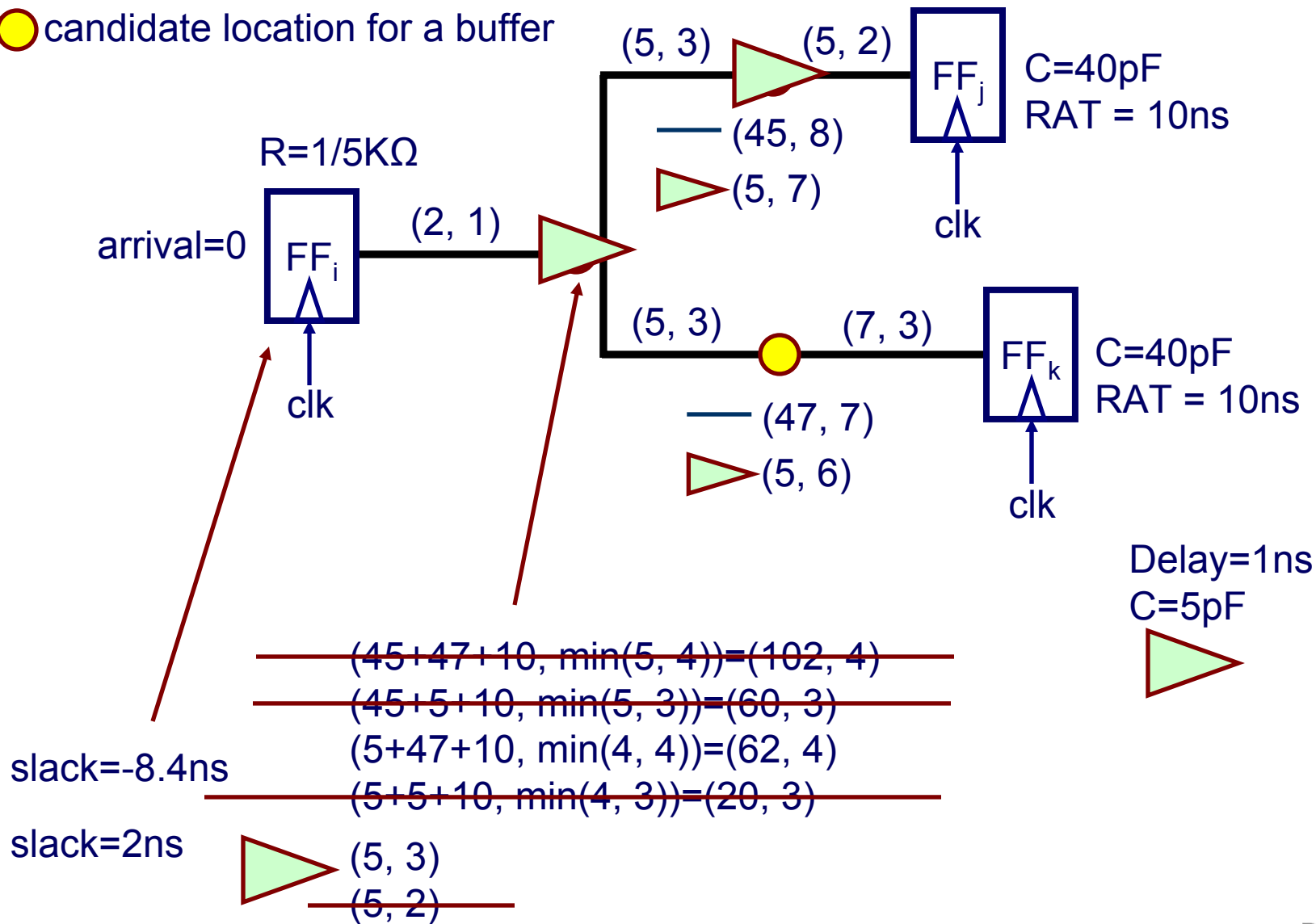


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# Van Ginneken algorithm

● candidate location for a buffer



slack=-8.4ns

slack=2ns



# Van Ginneken algorithm summary

- Bottom-up pass: calculate the various candidate solutions
- Top-down pass: select the winning solution
- Complexity?

# More questions

- How to minimize the number of buffers?
- How to determine simultaneously the tree topology and the buffers required?
- How to use inverters as well as buffers in repeater insertion?
- How to size a buffer?
- How to simultaneously size buffers and wires?
- .....
- Please check recommended readings



# Assigned Readings

- Optimal Wire Sizing and Buffer Insertion for Low Power and a Generalized Delay Model, DAC'95
- Interconnect Layout Optimization by Simultaneous Steiner Tree Construction and Buffer Insertion, ICCAD'96
- Minimum-Buffered Routing of Non-Critical Nets for Slew Rate and Reliability Control, ICCAD'01