EN2912C: Future Directions in Computing
Lecture 04: Fundamental Physical limits to Computing

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Limits to improving computing

A computer is an information processing engine that is ultimately constrained by the laws of physics.
Physical limitations

- Computers are physical systems. The laws of physics dictate their capabilities

- The laws of physics determine the ultimate limits for these crucial metrics:
  1. Switching energy limitations (this lecture)
  2. Switching time limitations (this lecture)
  3. Size limitations (this lecture)
  4. Thermal limitations (part this lecture/next lecture)
  5. Noise limitations (next lectures)
Main two ingredients for digital computing

Switches

interconnects

What is:
1. The minimum switching time?
2. The minimum switching energy?
3. The minimum device size? *irrespective of technology*

Minimum propagation delay

\[
\tau \geq \frac{L}{c_0}
\]

c_0 is the velocity of light in free space
1. Minimum energy required for a binary transition

If a force $F$ moves through a small distance $\delta x$, the work done $\delta W$ is:

$$\delta W = F \delta x$$

If the pressure of the gas is $p$, and the cross section of the piston is $A$ then

$$\delta W = P \delta V$$

From classical physics

$$pV = NkT$$

where $N$ is the number of particles in the gas and $k$ is Boltzmann's constant

$$W = \int_{V_1}^{V_2} \frac{NkT}{V} dV = NkT \ln \frac{V_2}{V_1}$$
1. Minimum energy for irreversible switch

- Consider that we have one particle
- If the particle is in the left half of the “box” then the switch is in state 0 and if the particle in the right half then the switch in state 1.
- Therefore the \textit{minimum energy per irreversible transition between two states} is equal to \( E_{\text{min}} = KT \ln 2 \)
1. Contrast to CMOS switching energy

• At 130 nm, $CV^2 = 0.17 \text{ fJ}$
• At 22 nm, $CV^2 = 0.005 \text{ fJ}$
• $KT\ln 2 = 3 \times 10^{-21} \text{ J} = 0.000003 \text{ fJ @ 300K}$

• We are still orders of magnitude above the theoretical $KT\ln 2$ limit!!
Quantum-mechanical limits: Uncertainty principle

A state which only exists for a short time cannot have a definite energy.

[Jeff Welser, IBM]
2. Fundamental limits on switching time

From Heisenberg uncertainty principle

\[ \Delta E \Delta t \geq \frac{h}{2\pi} \]

Minimum transition time between two states

\[ \Delta t_{\text{min}} \geq \frac{h}{2\pi E_{\text{min}}} \]

Maximum operation frequency

\[ F_{\text{max}} \leq \frac{2\pi E_{\text{min}}}{h} \]

\[ E_{\text{min}} = KT \ln 2 \]

\[ F_{\text{max}} \leq 25000 \text{GHz} \]
3. Minimum distance requirements

\[ \Delta x \Delta p \geq \hbar \]

\[ E_{\text{min}} = K T \ln 2 \]

\[ x_{\text{min}} = a = \frac{\hbar}{\sqrt{2mkT \ln 2}} = 1.5 \text{nm}(300K) \]

\[ n = \frac{1}{x_{\text{min}}^2} = 4.6 \times 10^{13} \frac{\text{gate}}{\text{cm}^2} \]

Gate density

[Jeff Welser, IBM]
Putting it together

Power consumption

\[ P = nE_{\text{min}}F_{\text{max}} \]

\[ = 4.6 \cdot 10^{13} \cdot 3 \cdot 10^{-21} \cdot 2.5 \cdot 10^{12} \]

\[ = 4.74 \times 10^6 \text{ W/cm}^2 \]

The circuit would immediately vaporize once it is turned on

*Circuit heat generation is the main limiting factor for scaling of device speed and switch circuit density*
Paper discussions