EN2912C: Future Directions in Computing
Lecture 05: Thermal Noise and Limits to Computing

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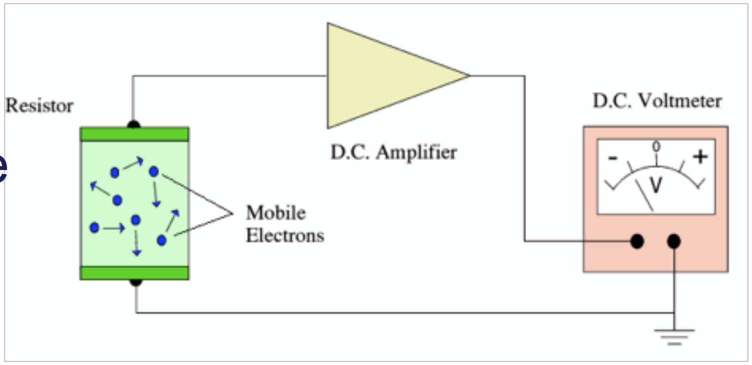
Noise Phenomena in Circuits

- Thermal (Johnson-Nyquist) noise
- Shot noise
- Flicker noise

Thermal (Johnson-Nyquist) Noise



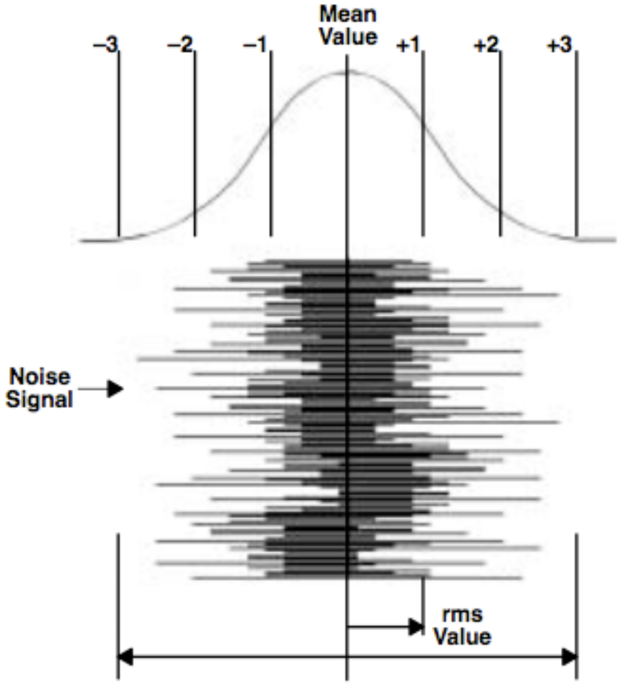
Thermal noise is the electronic noise generated by the thermal agitation of the charge carriers (usually the electrons) inside an electrical conductor at equilibrium, which happens regardless of any applied voltage.

Thermal noise is approximately white. Noise amplitude has very nearly a Gaussian probability density function.

Power spectral density or voltage variance is given by (double check R)

$$\overline{v_n^2} = 4KTR$$

K is the Boltzmann constant
T is the temperature
R is resistance



99.7% Probability Signal Will Be <= 6 X rms Value

Thermal (Johnson-Nyquist) Noise

For a bandwidth Δf, the root mean square (RMS) of the voltage is

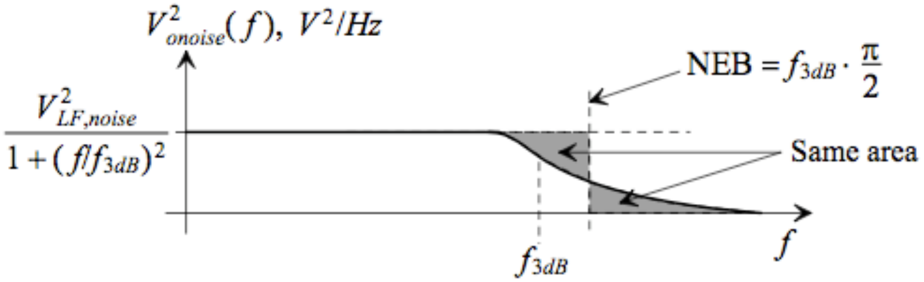
$$v_{n,RMS} = \sqrt{4KTR\Delta f}$$

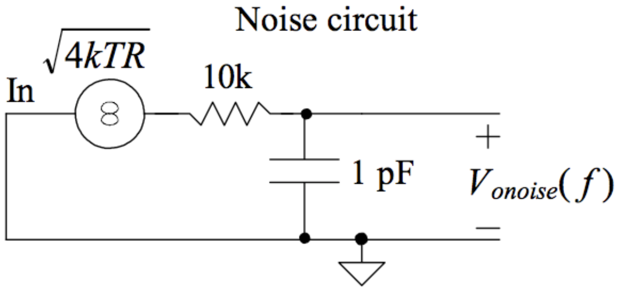
Example: for R = 1 kΩ at room temperature and a 1 GHz bandwidth, the RMS of the noise voltage is equal to 0.13 mV

Any resistor not connected to a voltage source will have a voltage associated with it. Note that the power delivered by the resistor only depends on its temperature. Max power transfer occurs when load = R

$$P = \frac{v_{out}^2}{R} = \frac{(v_{n,RMS}/2)^2}{R} = KT\Delta f$$

Thermal Noise in RC circuits





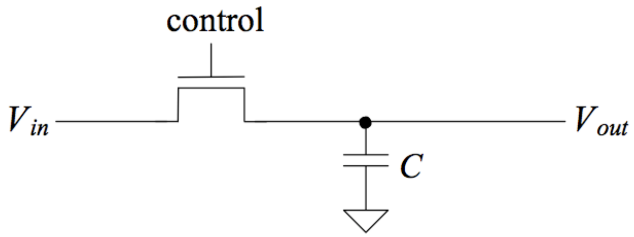
$$\begin{aligned}
Vonoise(f) &= \sqrt{4KTR}\frac{1/jwC}{1/jwC+R} \\
&= \frac{\sqrt{4KTR}}{1+jwRC} \\
V^2_{onoise,RMS} &= \int_0^\infty \frac{4KTR}{(\sqrt{1+(2\pi fRC)^2})^2}df \\
&= \int_0^\infty \frac{4KTR}{1+4\pi^2 f^2 R^2 C^2}df \\
V_{onoise,RMS} &= \sqrt{\frac{kT}{C}}
\end{aligned}$$

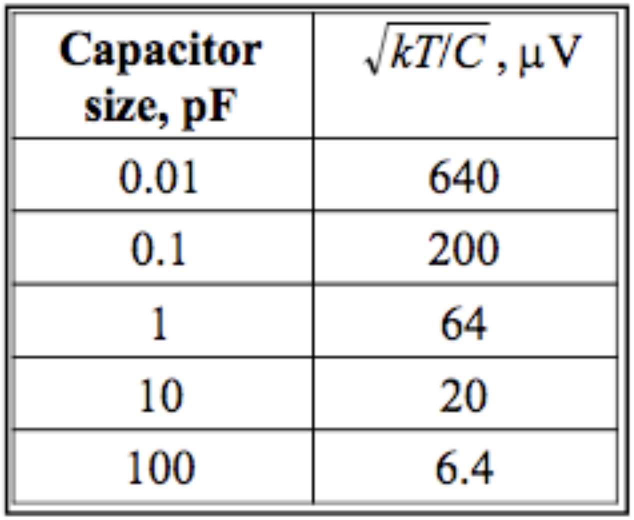
The RMS value is limited by the capacitor and independent of the size of the resistor

Thermal Noise in RC circuits

When the control transistor gate is driven to VDD, the transistor behaves like a resistor, and the capacitor samples both the input signal Vin and the KT/C noise onto the capacitor.

➡ Vin has to be much larger than the KT/C noise to be able to reliably distinguish the sampled voltage afterwards





Capacitor size, pF	$\sqrt{kT/C}$, μV
0.01	640
0.1	200
1	64
10	20
100	6.4

Paper discussions

- Ultimate Physical Limits to Computation, Lloyd, Nature 406, 1047-1054, 2000.
- End of Moore’s Law: Thermal (Noise) Death of Integration in Micro and Nano Electronics, Physics Letters A, 2002, 144-149.