EN2912C: Future Directions in Computing
Lecture 05: Thermal Noise and Limits to Computing

Prof. Sherief Reda
Division of Engineering
Brown University
Fall 2008
Noise Phenomena in Circuits

- Thermal (Johnson-Nyquist) noise
- Shot noise
- Flicker noise
Thermal (Johnson-Nyquist) Noise

Thermal noise is the electronic noise generated by the thermal agitation of the charge carriers (usually the electrons) inside an electrical conductor at equilibrium, which happens regardless of any applied voltage.

Thermal noise is approximately white. Noise amplitude has very nearly a Gaussian probability density function.

Power spectral density or voltage variance is given by (double check R)

\[ \overline{v_n^2} = 4KTR \]

K is the Boltzmann constant
T is the temperature
R is resistance

S. Reda. Brown U. EN2912C Fall ‘08
Thermal (Johnson-Nyquist) Noise

For a bandwidth $\Delta f$, the root mean square (RMS) of the voltage is

$$v_{n,RMS} = \sqrt{4kT R \Delta f}$$

Example: for $R = 1$ k$\Omega$ at room temperature and a 1 GHz bandwidth, the RMS of the noise voltage is equal to 0.13 mV

Any resistor not connected to a voltage source will have a voltage associated with it. Note that the power delivered by the resistor only depends on its temperature. Max power transfer occurs when $load = R$

$$P = \frac{v_{out}^2}{R} = \frac{(v_{n,RMS}/2)^2}{R} = kT \Delta f$$
Thermal Noise in RC circuits

\[ V_{\text{noise}}(f) = \sqrt{4kTR} \frac{1/jwC}{1/jwC + R} \]

\[ = \frac{\sqrt{4kTR}}{1 + jwRC} \]

\[ V_{\text{noise, RMS}}^2 = \int_0^\infty \frac{4kTR}{(\sqrt{1 + (2\pi f RC)^2})^2} df \]

\[ = \int_0^\infty \frac{4kTR}{1 + 4\pi^2 f^2 R^2 C^2} df \]

\[ V_{\text{noise, RMS}} = \sqrt{\frac{kT}{C}} \]

The RMS value is limited by the capacitor and independent of the size of the resistor
Thermal Noise in RC circuits

When the control transistor gate is driven to VDD, the transistor behaves like a resistor, and the capacitor samples both the input signal $V_{in}$ and the $KT/C$ noise onto the capacitor.

$V_{in}$ has to be much larger than the $KT/C$ noise to be able to reliably distinguish the sampled voltage afterwards.

<table>
<thead>
<tr>
<th>Capacitor size, pF</th>
<th>$\sqrt{kT/C}$, $\mu$V</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>640</td>
</tr>
<tr>
<td>0.1</td>
<td>200</td>
</tr>
<tr>
<td>1</td>
<td>64</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>100</td>
<td>6.4</td>
</tr>
</tbody>
</table>
Paper discussions
